Quantum Security for the Fiat-Shamir Transform

Based on

J. Don, S. Fehr, C. Majenz, C. Schaffner, "Security of the Fiat-Shamir Transformation in the Quantum Random Oracle Model." CRYPTO 2019, pp 356-383, and the "Picnic" submission.

February 11, 2020, NIST Postquantum Crypto Seminar

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(Not for public distribution.)

The Basics

- The Fiat-Shamir transform can be used to turn interactive proofs-of-knowledge into digital signature schemes.

- This paper shows that Fiat-Shamir is secure in the quantum random oracle model (QROM).

- They offer some tentative applications to NIST PQC candidates.

Fiat-Shamir in the Classical Context

The Random Oracle Model

A hash function is an (efficiently computable) function

$$h: \{0,1\}^n \to \{0,1\}^m$$

which behaves a lot like a random function.

In a security proof "in the random oracle model," each use of the hash function is replaced by a black box,

which chooses a random output for each new input.

Σ-Protocols

Three rounds:

1. Commit.
2. Challenge.
3. Verify.

Bob then checks that a predicate P (x, e, w) holds.

$\xrightarrow{x}$

e (uniformly random) $\longleftarrow$

$\xrightarrow[w]{}$

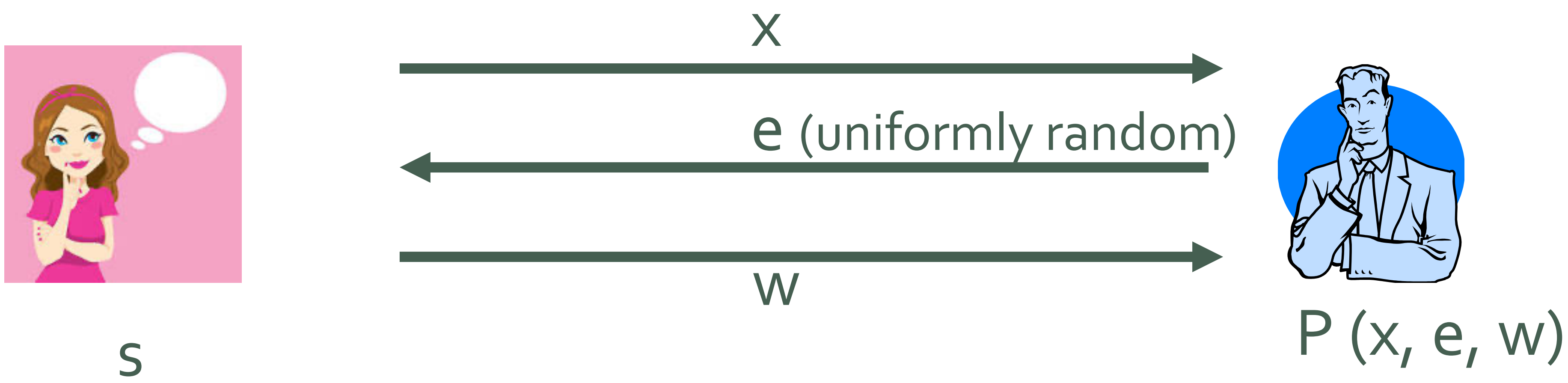
P (x, e, w)

Σ-Protocols

These are useful when there is a bit string s such that:

- Alice can efficiently satisfy P if she knows s;
- With no information, Alice can't efficiently satisfy P.

(Proof of knowledge.)



Fiat-Shamir Transform

Alice uses a hash function H to compute e.

s

$\xrightarrow{x}$

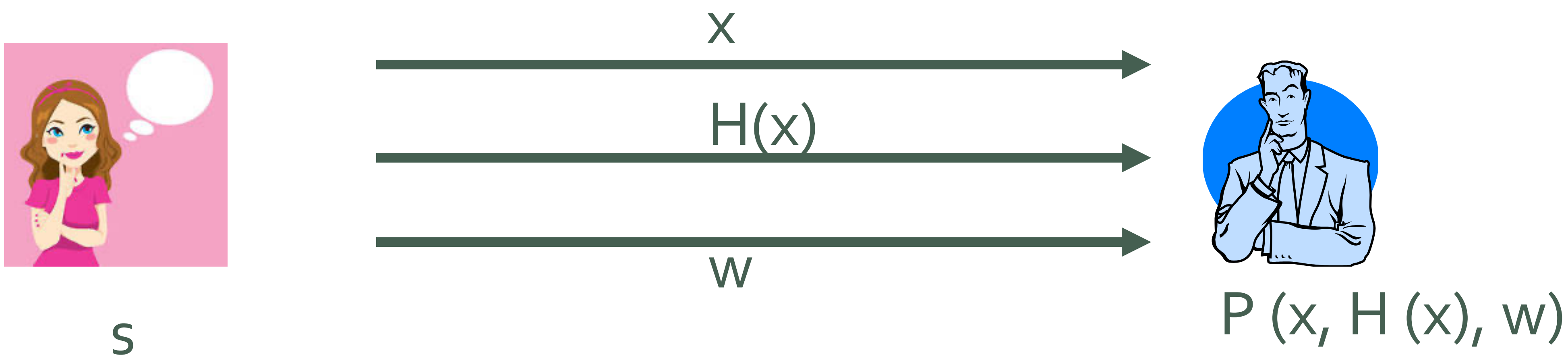
$\xrightarrow{e}$

$\xrightarrow[w]{}$

Fiat-Shamir Transform

Alice uses a hash function H to compute e.
Bob simply checks P on the hash.

(To make this a signature scheme, hash the message m as well.)



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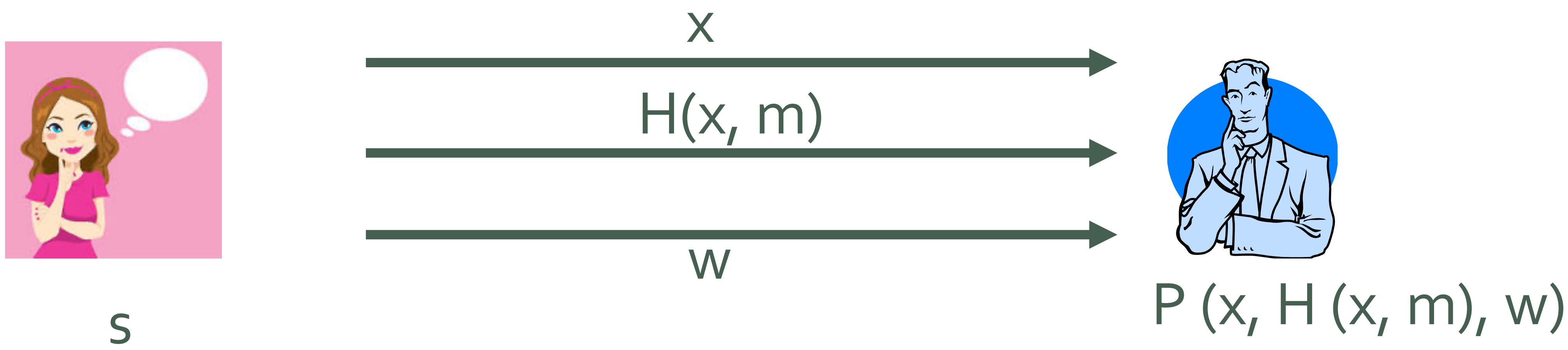
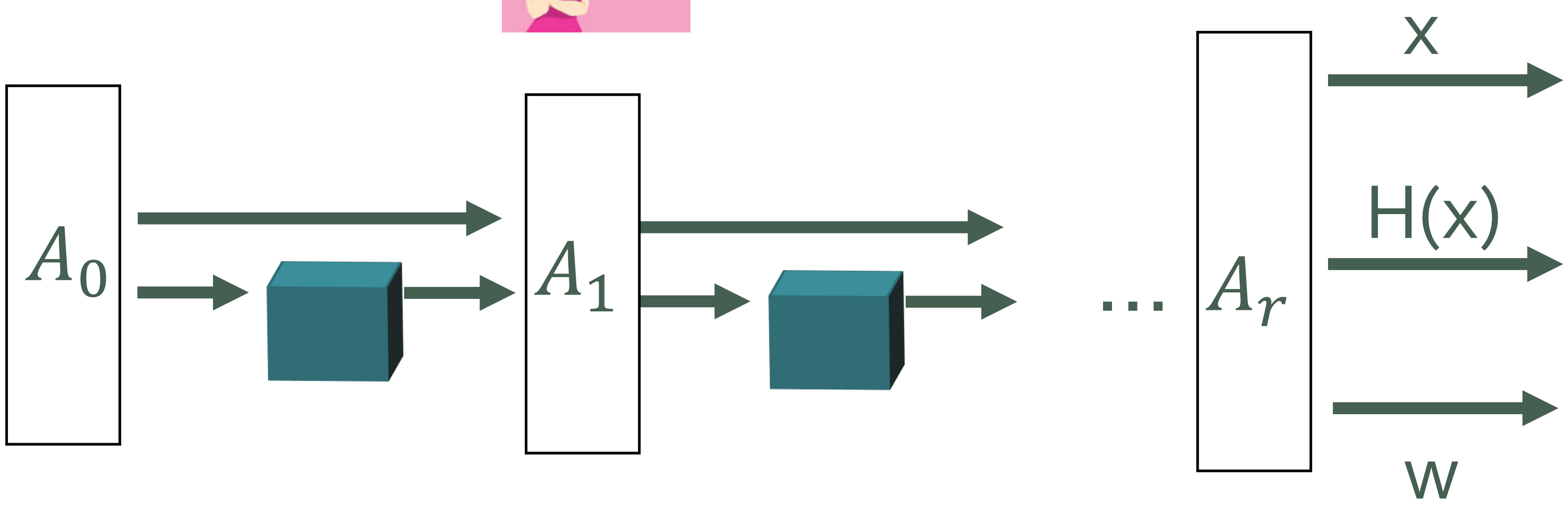


Fiat-Shamir is Secure in the ROM

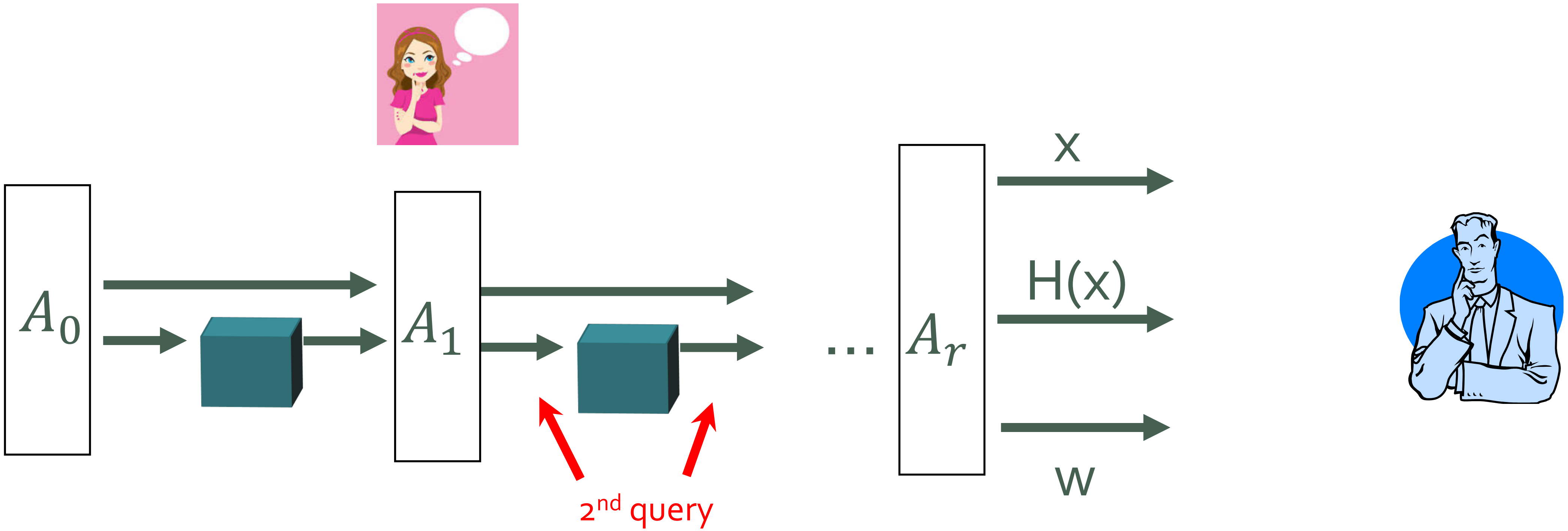
Suppose that Z is a Σ-protocol, and Alice has an algorithm that works for Fiat (Z) with non-negligible prob.
She wants an algorithm that works for Z.



Fiat-Shamir is Secure in the ROM

For i = 1, …, r, let p_i be the probability that the protocol succeeds **and** that the final pair (x, H(x)) was generated on the ith query.

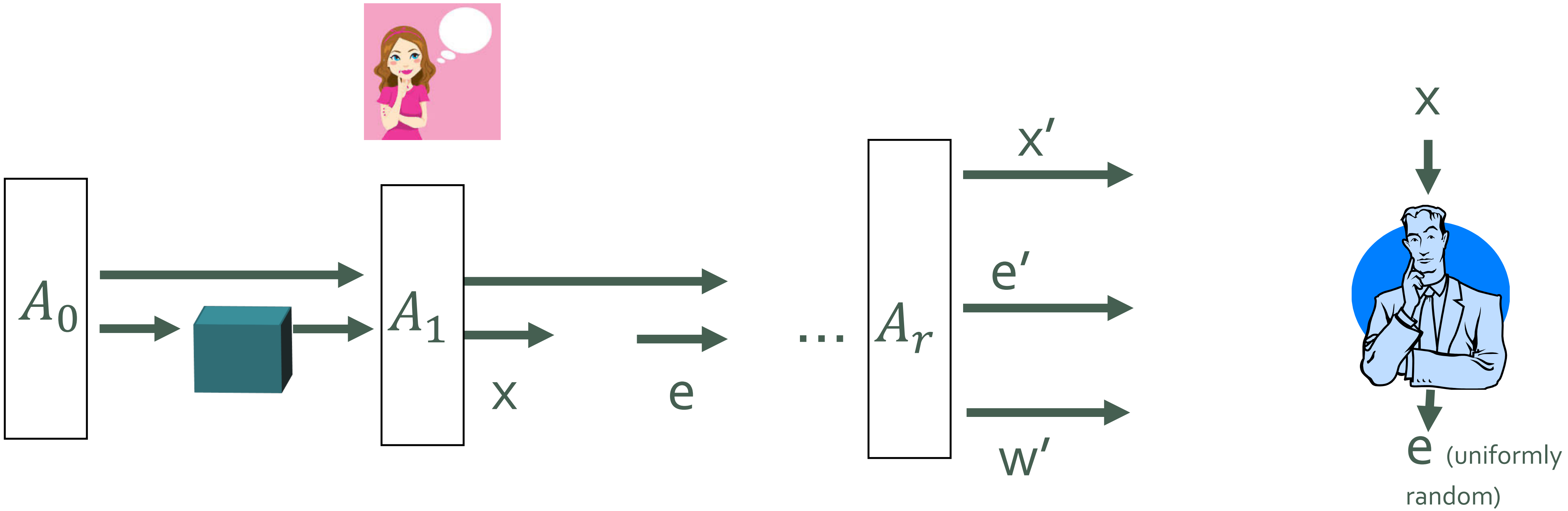
Let p = overall probability of success.



Fiat-Shamir is Secure in the ROM

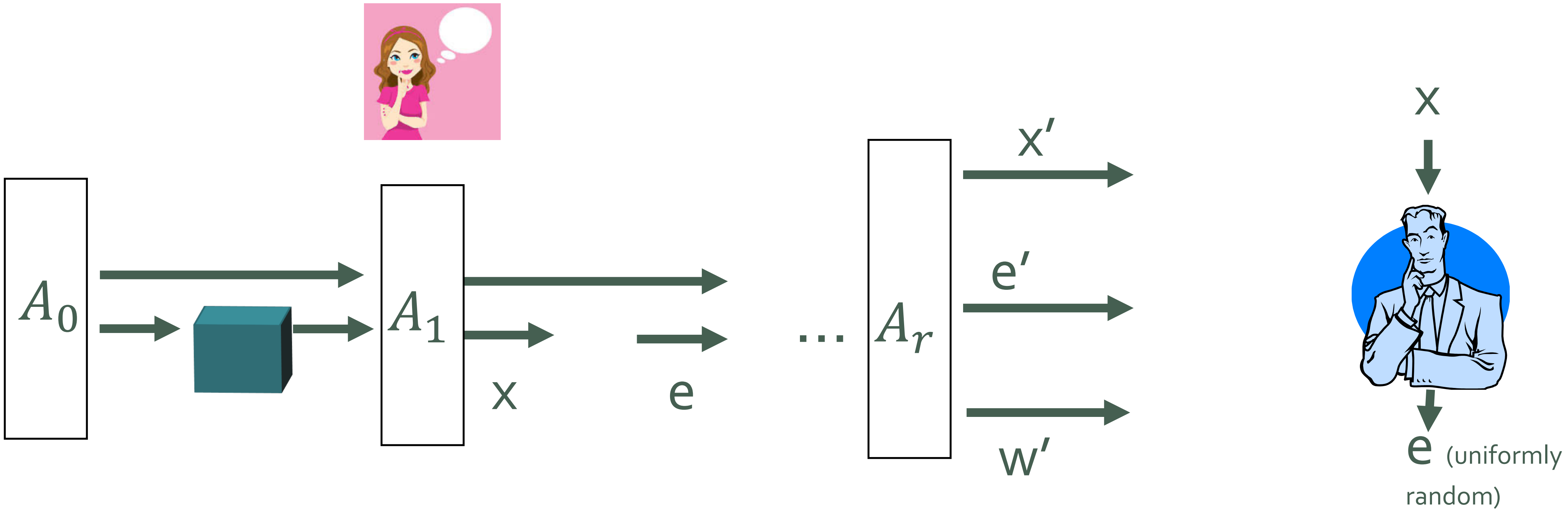
Alice chooses a random round i. On the ith round (only) she uses Bob in place of the random oracle.

She then finishes as usual.



Fiat-Shamir is Secure in the ROM

W/ prob. p_i, the output will include x and e as desired.

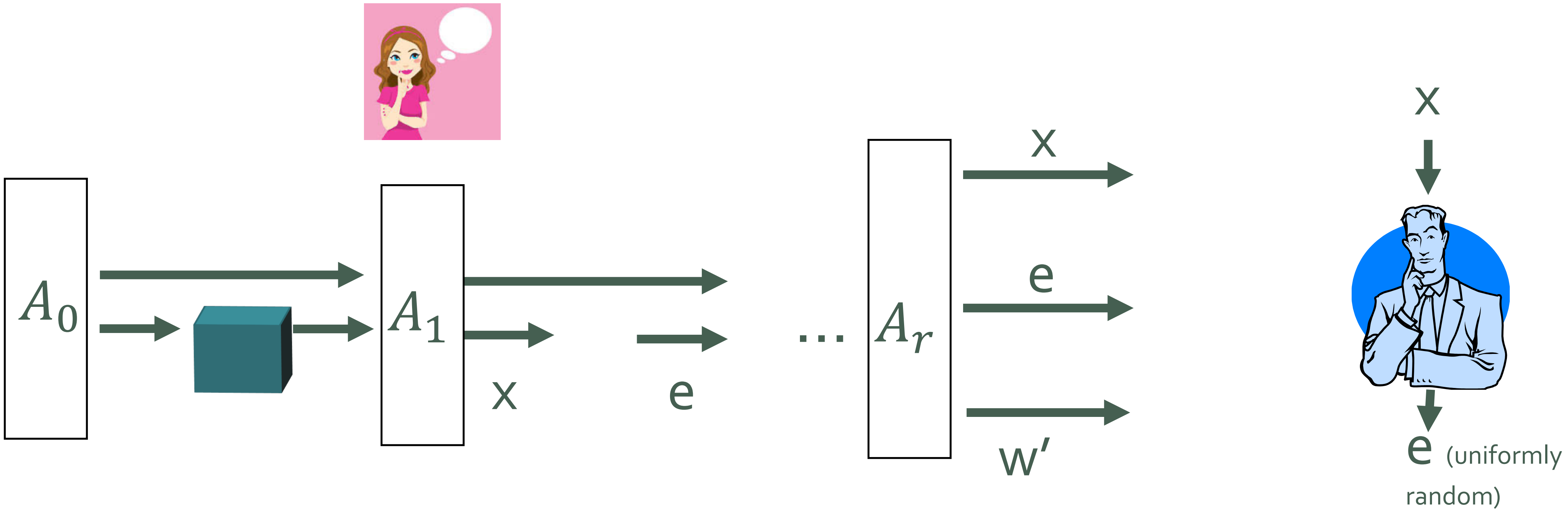


Fiat-Shamir is Secure in the ROM

W/ prob. p_i, the output will include x and e as desired.

So, the overall probability of success is

$$\frac{p_1+p_2+\cdots+p_r}{r} \approx \frac{p}{r}. \qquad \text{(non-negl.)}$$



The Quantum Random Oracle Model

The QROM

A quantum random oracle is initiated by choosing a random function

$$f: \{0,1\}^n \rightarrow \{0,1\}^m$$

The oracle accepts bit strings in superposition and returns outputs in superposition.

$$\frac{|a_1 > +|a_2 > +|a_3 >}{\sqrt{3}} \longrightarrow$$

$$\longrightarrow \frac{|a_1, f(a_1) > +|a_2, f(a_2) > +|a_3, f(a_3) >}{\sqrt{3}}$$

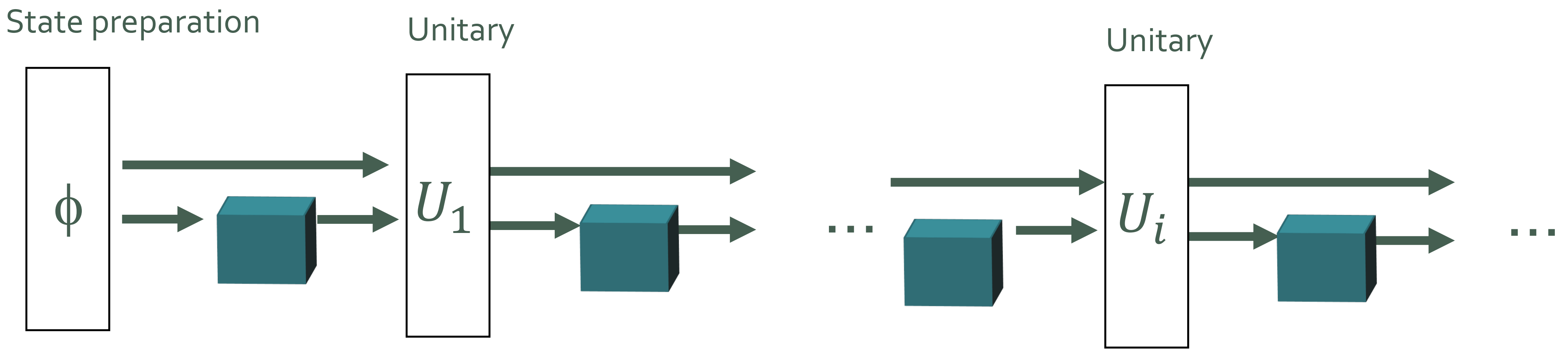
The QROM

Trying to adapt the previous Fiat-Shamir argument here raises multiple issues. (One is that measuring the input to a QROM disturbs it.)

We could use the **Unruh transform.**
That’s known to work, although it’s more complicated.

Fiat-Shamir is Secure in the QROM

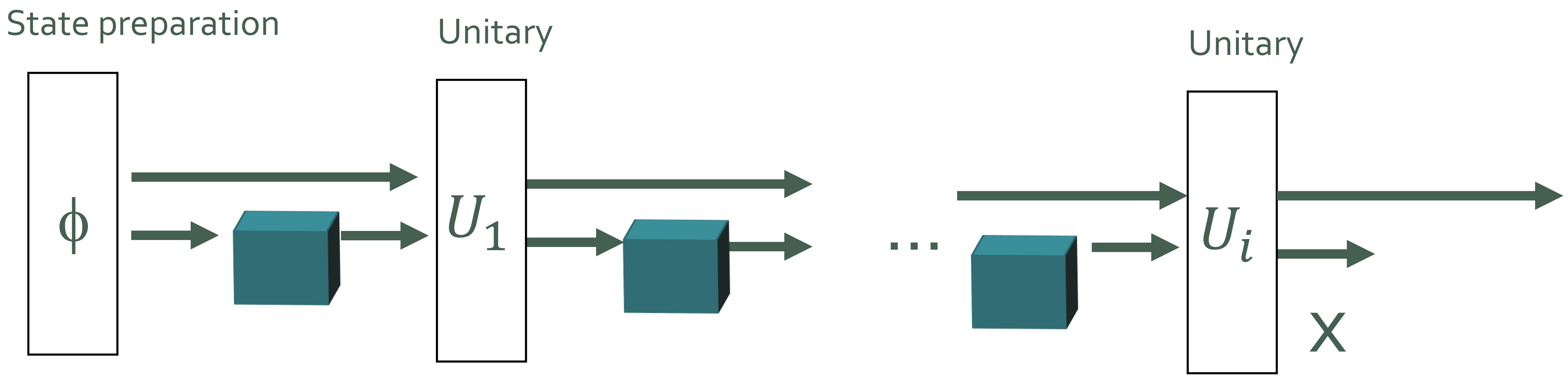
Suppose that Z is a Σ-protocol, and Alice has a **quantum algorithm** that works for Fiat (Z) with non-negligible prob.
She wants an algorithm that works for Z.



Fiat-Shamir is Secure in the QROM

Alice runs the protocol until a randomly chosen round i.

She measures x, sends it to Bob, receives e.

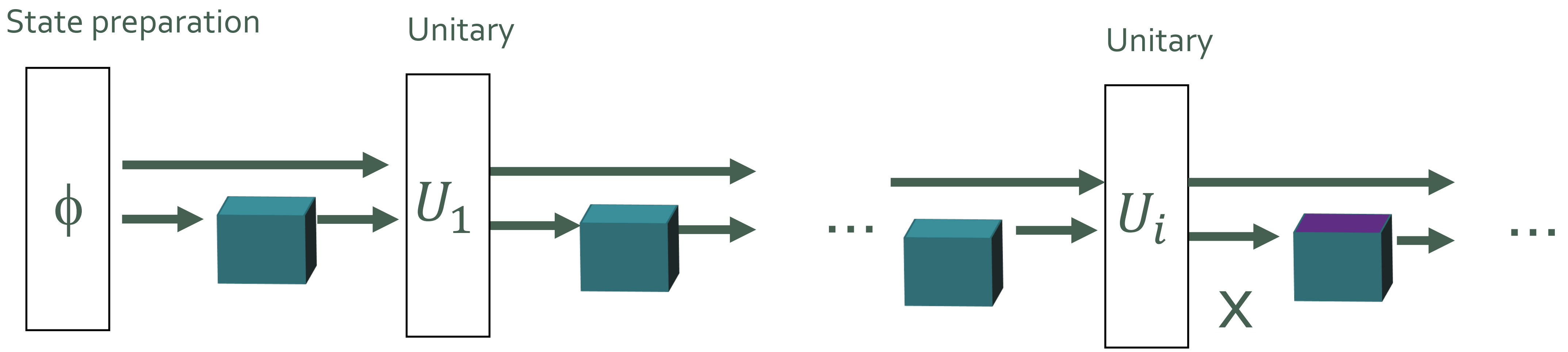


Fiat-Shamir is Secure in the QROM

She prepares an altered quantum oracle, forcing $x \to e$. 

She replaces the **<u>(i+2) thru rth</u>** uses of the oracle with the new one.

With probability ½, she replaces the **<u>(i+1)th</u>** use with the new one.



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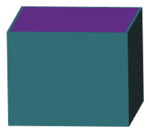
She prepares an altered quantum oracle, forcing $x \to e$. 

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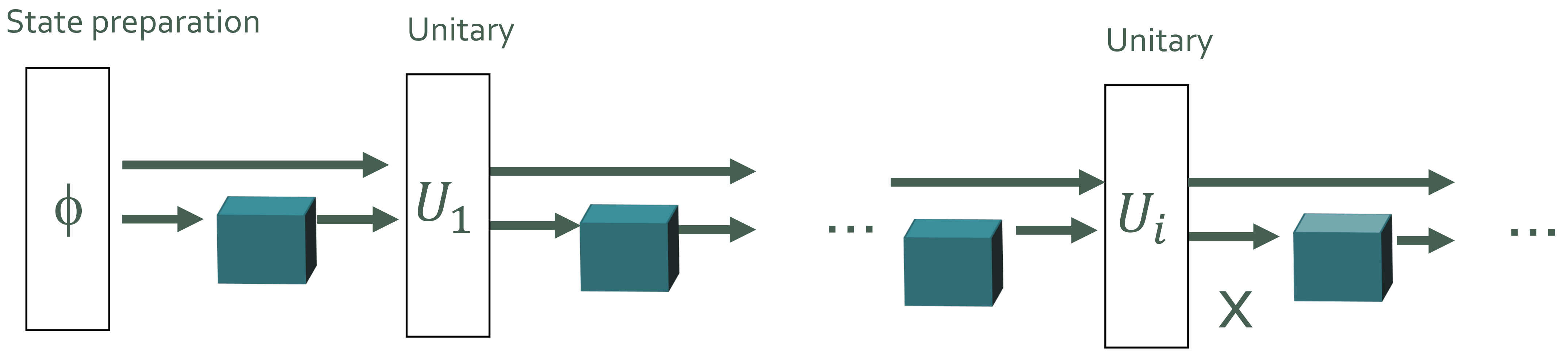

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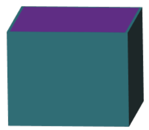
She prepares an altered quantum oracle, forcing $x \to e$. 

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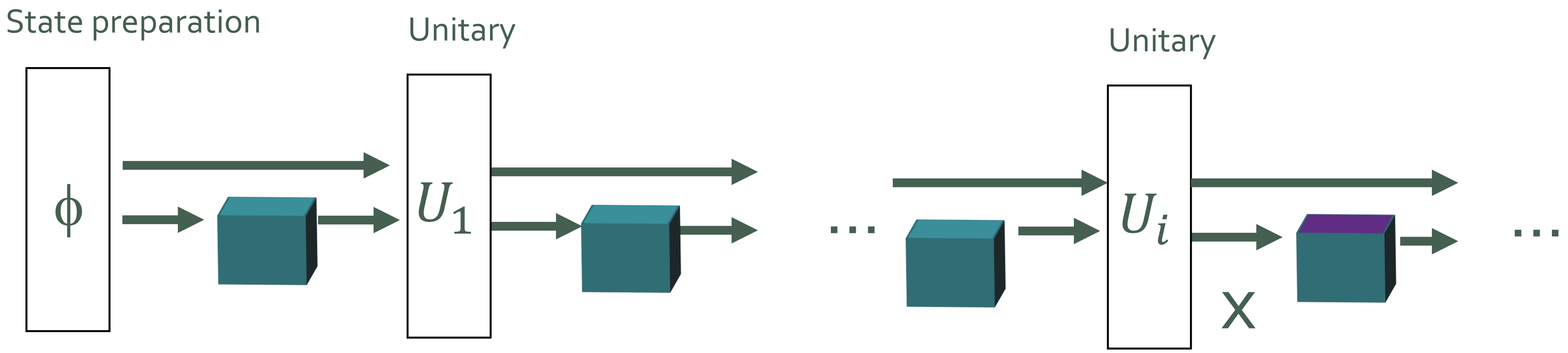
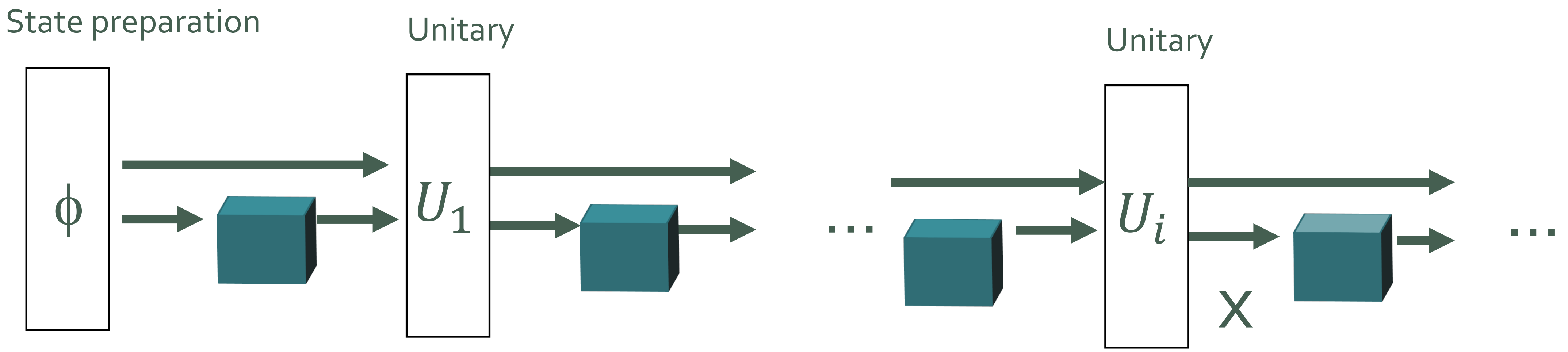


Fiat-Shamir is Secure in the QROM

Thm. With probability $\frac{p}{O(r^2)}$, the output will be of the form

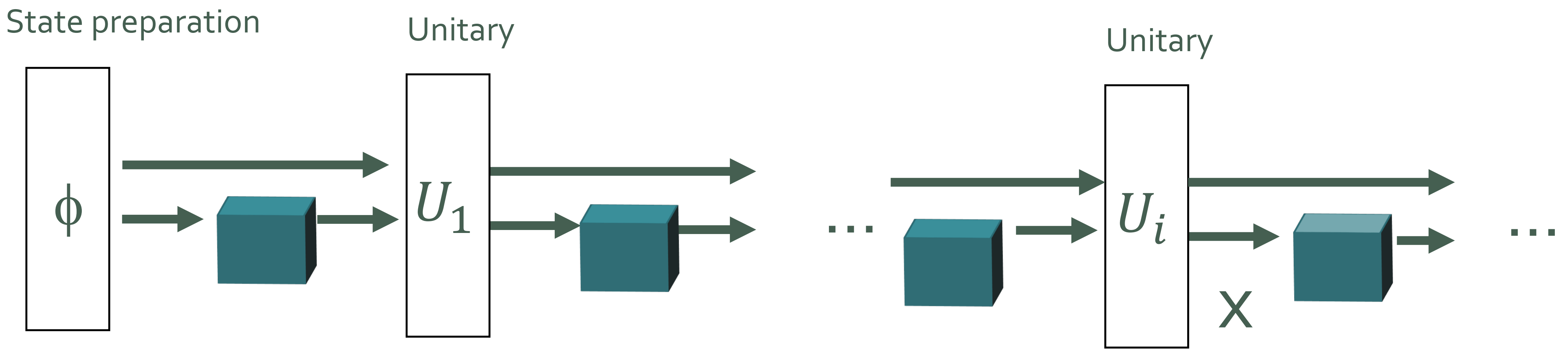
(x, e, w),

and the predicate P (x, e, w) will be satisfied.



Fiat-Shamir is Secure in the QROM

Conclusion: If Alice can win Fiat (Z) with non-negligible probability, then she can win Z with non-negligible probability.



Applications to PQC Candidates?

Picnic

In Picnic, the designers take two Σ-protocols (ZKB++ and KKW) and apply Fiat-Shamir and Unruh transforms to construct signatures schemes.

J. Don et al. explain a proof, via their main result, of a scheme similar to Picnic.

They also briefly address lattice-based schemes.

The Picnic Signature Scheme

Design Document

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